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1983 J. Phys. A: Math. Gen. 16 2161

(http://iopscience.iop.org/0305-4470/16/10/012)

J. Phys. A: Math. Gen. 16 (1983) 2161–2171. Printed in Great Britain

Quantum limits to the flow of information and entropy

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Received 15 October 1982

Abstract. The quantum limits to information flow are explored developing the central concept of a medium comprising several channels through which the information flows. In each channel there is an inequality between information flow $\dot{I}$ and energy flow $\dot{E}$

$$\dot{I}^2 \leq \dot{E}\pi/(3\hbar \ln^2 2)$$

a relationship which, though speculated on before or derived for only a restricted class of systems, is proven here for a wide range of systems. Applications are made to the energy cost of computing and to the maximum rate of cooling, $\dot{Q}$, which in any one channel is

$$\dot{Q} \leq \pi k_B^2 T^2/(3\hbar).$$

1. The medium is the message

When information is signalled from one place to another it must be conveyed by some medium. Despite information being an abstract concept, when we come to process it there is always a specific context: a flow of electric current, radio waves, light waves or sound waves. Identifying information with entropy (Shannon 1948, Brillouin 1956), there is an analogy to be made between a medium carrying information and a fluid which conveys entropy away from the interior of a refrigerator. This is the analogy we shall pursue. The medium will remain unchanged after the information transfer, but we shall require that it obeys the known laws of physics.

Already the laws of relativity tell us that information cannot travel faster than the speed of light. Recently there has been much interest in exploring whether quantum mechanics places further constraints on the transfer of information. Broadly speaking, work falls into two categories: the search for a very general law which holds under every conceivable circumstance and more limited studies dealing with specific systems. The work of Bremermann (1967) and Bekenstein (1981) falls into the former category. They find a relationship between information flow rate, $\dot{I}$, and the energy flow rate $\dot{E}$. However, the extreme difficulty of making a general argument leads to ambiguities and some scepticism has been expressed about the results (Deutsch 1982, Landauer 1981). On the other hand Lebedev and Levitin (1966) considered transmission of an electromagnetic field in one dimension, drawing the analogy with a single-channel communication system. They found an energy cost per bit of information depending on the flow rate in the same qualitative way that Bremermann's result does. Marko (1965) pointed out a relationship of this general form but without deriving the exact form of the coefficients. Further references may be found in the book by Yu (1976).

The general argument for energy bounds on information flow looks to the uncertainty principle between energy and time, and we reproduce it here even though it is not logically consistent as it stands, because it does show complete independence of the nature of the medium and perhaps points the way to a rigorous but completely general proof.

'If a receiver is to detect N bits of information in time t, energies cannot be resolved to within better than

$$\delta E \sim \hbar t^{-1}. \tag{1}$$

Suppose that information is conveyed by the arrival or non-arrival of a particle of given energy. To get N bits of information requires an energy spread $N\delta E$ and a mean energy per particle of

$$\tfrac{1}{2}N\hbar t^{-1}. \tag{2}$$

In order to detect a particle of a given energy, one quantum of energy must be received. Hence a flow of energy is unavoidable, and summing over N bands gives

$$\dot{E} \geqslant \tfrac{1}{2}\hbar N^2 t^{-2}. \tag{3}$$

At the same time the information flow rate is

$$\dot{I} = Nt^{-1} \tag{4}$$

hence

$$\dot{I}^2 \leqslant (\pi/3 \ln^2 2)\dot{E}\hbar^{-1} \tag{5}$$

where the precise form is quoted in anticipation of later results.'

This is a remarkable result and implies that the 'information wind' drags along some energy. There is not a fixed cost per bit, but an increasingly heavy energy price as the flow rate increases. According to formula (5) an information flow rate of 10^{17} Hz would require a power of at least 1 W.

Delightfully simple as this argument is, it is unfortunately seriously flawed. Leaving aside the questions of the precise nature of a receiver, whether 'detection' of a photon does mean capture and dissipation of its energy and how many bits can be conveyed per photon (Landauer 1982), I turn to what I believe to be the essential missing ingredient in the argument, which is the question of parallelism or the number of channels open for communication. Suppose that the receiver was connected to a single optical fibre. Then we know from Lebedev and Levitin that the optimum flow rate

$$\dot{I}_1^2 = (\pi/3 \ln^2 2)\dot{E}_1\hbar^{-1} \tag{6}$$

is possible. A second fibre plugged into the receiver obeys a similar law. Since information and energy are additive quantities the capacity of the two lines together is

$$\dot{I}_2^2 \leqslant 2(\pi\dot{E}\hbar^{-1}/3 \ln^2 2), \tag{7}$$

which at optimum breaks our new law. By placing as many channels in parallel as we wish, we can arbitrarily reduce the energy expenditure for information transfer. This is the argument of Landauer and Woo (1973). It leaves us with no alternative but to concentrate on a single channel if we are to establish equation (5) on a sound footing.

When information arrives conveniently confined in an optical fibre, defining a channel presents no problem. In more complex systems, especially when we wish to include entropy in our argument, it is not so easy. Suppose that the receiver is a finite object totally enclosed within a surface of finite area A. To specify completely the medium transporting energy and information into the receiver requires several variables. For an electromagnetic wave incident on a planar surface there would be the polarisation and the momentum parallel to the surface which takes discrete values with a density

$$A(2\pi\hbar)^{-2} \tag{8}$$

in momentum space.

Each of these parallel momentum/polarisation states can be used as a separate channel and in the case of photons there is an infinite number of channels. Additional arguments can be raised to the effect that a certain minimum energy is associated with each parallel component of photon momentum, but these arguments are specific to photons. The minimum energy would be different for sound waves and different again for electrons.

Thus we define a single channel as corresponding to one of the variables needed to specify the flow of the medium into the receiver. By confining our attention to a single channel we sidestep all the difficulties of parallelism and can still hope to retrieve our basic inequality but now firmly confined to one channel.

In the real world a receiver has a finite surface area and therefore the restriction of our argument to a single channel would appear to render it obscure if not irrelevant. In fact this is not a serious restriction because receivers (radios, televisions, computers) consist, at the most, of a finite number of sequential processors each of which can handle only one channel. Thus a conducting wire carrying information from a computer store to the processor has in principle many millions of parallel channels. In practice it is impossible to do anything other than strap all these channels together and send the same message down each, whereupon the system reduces to a single effective channel. Thus in all circumstances where we are dealing with information the number of channels is known and is finite.

In contrast the flow of entropy down a wire in the form of heat conduction can make use of all the parallel channels and to make any sense of the inequality for a single channel, a counting exercise must be performed. As explained, this will depend on the detailed properties of the medium. Even here we can get useful results which are surprisingly independent of the details of medium structure. As an example, take a flow of entropy conducted by phonons, as it is for an insulating crystal. Here we can count the number of channels quite easily: it is given by

$$3n_{\mathrm{A}}N_{\mathrm{c}} \tag{9}$$

where n_{A} is the number of atoms per unit cell and N_{c} is the number of unit two-dimensional cells of the crystal lattice projected onto the plane perpendicular to the direction of flow.

Another example of simple channel counting occurs in the cooling of small objects. At sufficiently low temperatures the wavelength of the particles effecting the cooling will become long, and unable to distinguish the structure of the object being cooled which now becomes equivalent to a point. The number of variables needed to specify communication with a point is finite. For example, electromagnetic waves radiating entropy from a small object *in vacuo* couple only to the three components of the object's dipole moment. Therefore, only three channels operate at low temperatures. Gravitational waves cooling a star have five channels at low temperatures.

In § 2 the properties of single channels are investigated under conditions of wide generality which apply in most common cases of information and entropy flow. The proof makes no mention of the time/energy uncertainty principle and instead considers a steady flow of information. Thus within the confines of the model it is, I believe, rigorous. Since the same law emerges under a wide range of circumstances it is tempting to speculate that it is a law of quantum thermodynamics, the 'fourth law', unproven in the general case but universally obeyed.

2. Information-energy flow inequalities

In most common systems in which information or entropy flows, the active medium is a gas of independent particles. This is certainly true for electromagnetic radiation, but even in solids photons, phonons, electrons and holes play the role of a gas of independent particles. Other complications involving particle–particle interaction, such as magnetism, often produce their own series of excitations.

Two major factors classify the free gas:

(i) the type of statistics obeyed;

(ii) the dispersion relationship, $\omega(k)$, of the particles.

The important result of this section is that neither the statistics nor the dispersion relationship affects the inequality, though certain forms of $\omega(k)$ may render the optimum case unattainable.

We shall assume that a channel has been selected, for example, in the case of particles flowing across a plane, by fixing the momentum parallel to the plane, or by fixing the angular momentum when the particles flow out of a sphere. We further assume that a quantum number k can be found such that the particle velocity in the channel is

$$v(\omega) = \mathrm{d}\omega/\mathrm{d}k. \tag{10}$$

In free space k would be the momentum normal to the plane across which the particles flow. However, the assumption that k exists is not a trivial one and implies a symmetry to the transporting medium. If we take the case of electrons in a crystal, which has only discrete translational symmetry, equation (10) for the velocity exists only outside any 'band gaps' in the $\omega(k)$ function. In fact it is sufficient that $v(\omega)$ is defined only in a piecewise fashion for our proof to hold, except that the optimum case will not be attained. More serious difficulties arise for transport through a system which has no translational symmetry, for example in a disordered system. Here the particle motion is diffusive rather than ballistic, at least where the disorder is weak enough to permit diffusion. A rigorous quantum mechanical treatment of diffusion is beyond present techniques, but it would be surprising if processes which actually impede transport resulted in a more energy efficient system than obtained in the ordered case. Further, we can prove to some extent that the disordered case will be less efficient by representing it as the limiting case of an ordered system with long periodicity.

Having established the concept of velocity of the particles, we deduce from time reversal that there are two states of the particle in the channel corresponding to $\pm v$, each having its own source on opposite sides of the medium. The strategy is to push the maximum amount of entropy away from a source for a given amount of energy flowing from this source. We shall label the entropy and energy flows as $\dot{S}_+$ and $\dot{E}_+$ where the subscript '+' denotes that the flux originates from the '+' source. These subscripts are important because it is always possible to cancel the energy flow from one source with a counterflow in the opposite direction, but still leave the system with a net entropy flow. Thus our work does not imply an inevitable energy expenditure, a conclusion which would require a detailed analysis of transmitter and receiver and which will not be attempted here. The work in recent years on reversible computation (Benioff 1982, Landauer 1982) points to the need for careful re-examination of the prevalent assumption that the energy in a message must be dissipated.

Maximising the entropy flow implies that the velocity distribution must be black-body like as if the particles had been emitted from a surface which is a perfect absorber of particles. That a black-body distribution gives the maximum throughput of entropy can be seen as follows. Consider a channel formed into a ring so that the particles circulate continuously. The number of particles and the total energy is fixed, but the entropy need not be, and any coupling between the particles making accessible all states with *positive* velocities will lead to irreversible changes and to an increase in entropy. Eventually the entropy will reach a maximum and thermal equilibrium will be established in the channel characterised by T_c and a black-body distribution. This follows from classical statistical mechanics. At the same time the entropy flow is maximised: suppose the interactions between the particles were confined to a short segment of the circle. Since energy is conserved the flow into this segment equals the flow out and therefore energy flow is conserved as is the particle flow. However, the entropy flow is not conserved but can only increase monotonically until thermal equilibrium is established. Thus the maximum in S_+ coincides with the maximum in $\dot{S}_+$.

In the case of information flow, T_c is the information temperature and we shall assume that the medium is isolated from its surroundings. Thus we do not consider the case of classical information theory where the information competes with entropy from thermal excitations.

Consider first the case of fermions. In the first instance we assume that $\omega(k)$ is single valued and continuous from $\omega = -\infty$ to $+\infty$, i.e. that the Fermi energy lies in a band and that any thermal excitations do not reach the band edges. We shall ultimately argue that the existence of band gaps only reduces the amount of entropy transported.

The state of the channel medium is specified by a flow rate of particles which must be a constant at all points on the channel in the steady state

$$\dot{n} = \text{constant} = \int_{-\infty}^{\infty} \frac{1}{\exp[(\hbar\omega - \mu)/k_B T] + 1} \frac{d\omega}{2\pi}. \tag{11}$$

Note that the group velocity and the density of states both occur in the formula and factors of $d\omega/dk$ cancel. This is a key to the universality of the result. The constancy of $\dot{n}$ implies that μ is the same at all points in the system and independent of temperature. The flow of energy down the channel is

$$\dot{E}_+ = \int_{-\infty}^{\infty} \frac{1}{\exp[(\hbar\omega - \mu)/k_B T] + 1} \frac{d\omega}{2\pi} \tag{12}$$

where we count both hole and electron energies. Again the velocity cancels with the density of states. We can reduce this expression to

$$\dot{E}_+ = \frac{k_B^2 T_c^2}{\pi\hbar}\int_0^\infty \frac{x\,dx}{e^x+1}. \tag{13}$$

The entropy flow is calculated by integrating the heat content of the gas up to temperature T_c:

$$\dot{S}_+ = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_0^{T_c}\frac{dT}{T}\frac{d}{dT}\frac{|\hbar\omega-\mu|}{\exp(|\hbar\omega-\mu|/k_BT)+1}$$

$$= \frac{2k_B^2 T_c}{\pi\hbar}\int_0^\infty \frac{x\,dx}{e^x+1}. \tag{14}$$

Eliminating temperature from these expressions and recognising that we have calculated the minimum energy

$$\dot{S}_+^2 \le \frac{4k_B^2\dot{E}_+}{\pi\hbar}\int_0^\infty \frac{x\,dx}{e^x+1} = \frac{\pi k_B^2}{3\hbar}\dot{E}_+ \tag{15}$$

or in terms of information flow

$$\dot{I}_+^2 \le (\pi/3\ln^2 2)\dot{E}_+\hbar^{-1}, \tag{16}$$

a relationship which is completely independent of the form of $\omega(k)$ or any other physical parameter of the specific system.

In the case that $\omega(k)$ has gaps where $v(\omega)$ is not defined, then we argue that if we try to occupy states at fixed entropy flow, the missing ranges force us to use higher energy states where lower ones would have been used in the absence of gaps. Hence the energy flow is inevitably greater.

Realisations of this system might involve the ballistic transport of electrons through thin metallic components in a computer. Most devices are wasteful of energy because they effectively send the same information down many channels simultaneously. Typically if we wish to send information at $\sim 10^9$ bits per second then an energy flow of

$$\dot{E}_+ \ge 10^{-16}\,\mathrm{W} \tag{17}$$

must accompany the information. Clearly we are far from even noticing this amount of energy in current generations of computers.

On the other hand, ballistic transport in an intrinsic semiconductor is much less efficient because of the gap. Assuming the Fermi energy to be in the centre of the gap and reworking equations (13) and (14) gives

$$\dot{E}_+ \ge \frac{k_B^2 T_c^2}{\pi\hbar}\int_{E_G/2k_BT_c}^{0}\frac{x\,dx}{e^x+1} \tag{18}$$

$$\dot{S}_+ = \frac{2k_B^2 T_c}{\pi\hbar}\int_{E_G/2k_BT_c}^{\infty}\frac{x\,dx}{e^x+1}. \tag{19}$$

If $E_G \gg k_BT_c$ the resulting inequality is

$$\dot{E}_+ \ge \frac{\dot{S}_+E_G}{4k_B\ln(k_BE_G/\pi\hbar\dot{S}_+)} = \frac{\dot{I}_+E_G\ln 2}{4\ln(E_G/\pi\hbar\dot{I}_+\ln 2)}. \tag{20}$$

Equation (20) is special to the case of an intrinsic semiconductor which in any case obeys the less restrictive but more general inequality (16). For a gap of 2 eV and information flowing at $\sim 10^9$ bits per second

$$\dot{E}_+ \geq 10^{-11}\ \mathrm{W} \tag{21}$$

which is still a very small amount of energy.

One important point to note is the incorrectness of the assumption that each particle carries one bit of information. For an intrinsic semiconductor this would imply

$$\dot{E}_+ \geq \dot{I}_+ \tfrac{1}{2} E_{\mathrm{G}} \tag{22}$$

since each particle (hole or electron) has an excitation energy of $\frac{1}{2}E_{\mathrm{G}}$. This result is in clear contradiction with the more rigorously derived equation (20).

Next we consider a gas of bosons. In this case we assume, in the first instance, a spectrum $\omega(k)$ which is single valued and extends from $\omega = 0$ to $+\infty$. We argue that discontinuous spectra, as for fermions, result in greater energy flux. A spectrum extending to $\omega = -\infty$ has no meaning for bosons because all the particles could cascade into the lowest state for any finite μ. The chemical potential is $\mu = 0$ for massless bosons such as photons or phonons, but for massive particles we again demand that there be no accumulation of particles.

$$\dot{n}_+ = \text{constant} = \int_0^\infty \frac{1}{\exp[(\hbar\omega - \mu)/k_{\mathrm{B}} T_{\mathrm{c}}] - 1} \frac{\mathrm{d}\omega}{2\pi}. \tag{23}$$

In this case μ is not constant, but is always negative and decreases with increasing T_{c}. As before the energy flux is

$$\dot{E}_+ = \int_0^\infty \frac{\hbar\omega}{\exp[(\hbar\omega - \mu)/k_{\mathrm{B}} T_{\mathrm{c}}] - 1} \frac{\mathrm{d}\omega}{2\pi} = \frac{k_{\mathrm{B}}^2 T_{\mathrm{c}}^2}{2\pi\hbar} \int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^{x+x_0} - 1}, \tag{24}$$

where

$$x_0 = -\mu / k_{\mathrm{B}} T_{\mathrm{c}} \tag{25}$$

is a positive number. Again we write for the entropy flow

$$\dot{S}_+ = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \int_0^{T_{\mathrm{c}}} \frac{\mathrm{d}T}{T} \frac{\mathrm{d}}{\mathrm{d}T} \frac{\hbar\omega}{\exp[(\hbar\omega - \mu)/k_{\mathrm{B}} T_{\mathrm{c}}] - 1} = \frac{k_{\mathrm{B}}^2 T_{\mathrm{c}}}{\pi\hbar} \left(\int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^{x+x_0} - 1} - \gamma \right) \tag{26}$$

where

$$\gamma = \frac{k_{\mathrm{B}}^2}{2\pi\hbar} \left(-\int_0^{T_{\mathrm{c}}} \mathrm{d}T \int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^{x+x_0} - 1} + T_{\mathrm{c}} \int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^{x+x_0} - 1} \right) \tag{27}$$

is a positive number which corrects for the temperature dependence of x_0. Removing the factor of T_c gives

$$\dot{S}_+^2 \leqslant \frac{2k_B^2\dot{E}_+}{\pi\hbar}\left(\int_0^\infty \frac{x\,dx}{e^{x+x_0}-1}-\gamma\right)^2\left(\int_0^\infty \frac{x\,dx}{e^{x+x_0}-1}\right)^{-1}$$

$$\leqslant \frac{2k_B^2\dot{E}_+}{\pi\hbar}\int_0^\infty \frac{x\,dx}{e^x-1} \tag{28}$$

or

$$\dot{S}_+^2 \leqslant \tfrac{1}{3}k_B^2\pi\hbar^{-1}\dot{E}_+, \qquad \dot{I}_+^2 \leqslant \pi\dot{E}_+/3\hbar\ln^2 2 \tag{29}$$

which is, remarkably, the same inequality obtained for fermions: the equality is not attainable with massive bosons, but only with massless objects such as photons or phonons.

In the case that the spectrum does not extend to $\omega = 0$, the system is less efficient. For example, in an optical fibre there is a lower cut-off frequency for transmission associated with the energy needed to confine the light in the fibre. The effect is similar to the case of fermions where the Fermi energy lies in a gap. Suppose that the minimum energy for the boson spectrum is $\frac{1}{2}E_G$; then we deduce for this special case a more restrictive inequality

$$\dot{E}_+ \geqslant \dot{S}_+\frac{E_G}{2k_B\ln(k_BE_G/4\pi\hbar\dot{S}_+)} \tag{30}$$

or

$$\dot{E}_+ \geqslant \dot{I}_+\frac{E_G\ln 2}{2\ln(E_G/4\pi\hbar\dot{I}_+\ln 2)} \tag{31}$$

which is almost the same inequality as for fermions in an intrinsic semiconductor. Note again that much less energy is needed than would be implied by assuming one photon per bit of information.

If we assume that the cut-off energy for propagation in the fibre is about 1 eV, then at a transmission rate of 10^9 bits per second an energy flow of at least 10^{-11} W is required.

These bounds show that present systems for transporting information are very far from quantum limits. On the other hand, it is not hard to find in everyday life volumes of information that would produce very severe energy problems if forced through a single channel. Suppose we set out to send down a *single channel* all the information to make an exact copy of a volume of water at room temperature, after processing the material at the rate of 1 cm^3 per second. The enterprise would consume power at a rate of 10^{12} W. Alternatively the information could be conveyed by transporting the water itself from transmitter to receiver and this would consume negligible energy. There is no paradox because water generally flows from one point to another along millions of parallel quantum channels and the information flow in any one channel is small.

3. Quantum limits on rates of heat flow

We consider a system that is finite in size and isolated except for contact with a reservoir at a lower temperature. The contact might be through a metal (electrons) or an insulator (phonons) by radiating into a vacuum or by transmission of heat through a gas. In general, there will be many channels available but, in the first instance, we consider only one of them. If the system loses heat at a rate $\dot{Q}_+$, its entropy also decreases at a rate $\dot{Q}_+/T$. In a reversible process the whole of this entropy is transferred to the conducting medium, but if the process is irreversible the medium must carry away more entropy:

$$\dot{S}_+ \geqslant \dot{Q}_+/T. \tag{32}$$

If no external sources work on the system, $\dot{Q}_+$ is to be identified with the energy flow in the medium and substitution into the fundamental inequality (15) gives

$$\dot{S}_+^2 \leqslant \tfrac{1}{3}\pi k_B^2 \hbar^{-1} \dot{Q}_+ \leqslant \tfrac{1}{3}\pi k_B^2 T \hbar^{-1} \dot{S}_+ \tag{33}$$

hence

$$\dot{S}_+ \leqslant \tfrac{1}{3}\pi k_B^2 T \hbar^{-1} \tag{34}$$

or

$$\dot{Q}_+ \leqslant \tfrac{1}{3}\pi k_B^2 T^2 \hbar^{-1}. \tag{35}$$

These inequalities hold only for outward flow of heat and in the absence of return flow represent maximum rates of cooling.

It is clear why a channel should be limited in capacity by the temperature of the source. Equation (14) shows that entropy flow is limited by the effective channel temperature, T_c. At optimum

$$\dot{S}_+ = \tfrac{1}{6}\pi k_B^2 T_c \hbar^{-1} \qquad \dot{E}_+ = \tfrac{1}{12}\pi k_B^2 T_c^2 \hbar^{-1}. \tag{36}$$

Note that T_c is not the same as the system temperature. The optimum flow of entropy is obtained when a heat engine in the system pumps heat into the channel without employing external work, but using the *incoming* fluid in the channel, which at optimum has $T_c = 0$, as a sink for heat energy. If the heat were transferred to the channel by irreversible thermal contact with the cold incoming fluid only one *quarter* of the optimum heat flow would take place, because

$$T_c = T = \tfrac{1}{2}T_c(\text{optimum}). \tag{37}$$

Although the inequalities apply only to individual channels, there are certain circumstances under which we can count the number of channels available and obtain a global inequality for the total heat and entropy flows. One such instance is that of very low temperatures when all but a finite number of channels will be frozen out as the wavelength of the relevant cooling fluid becomes larger than the dimensions of the system being cooled. For example, at high temperatures a system cooling *in vacuo* obeys Stefan's law and the maximum cooling rate scales as (surface area) $\times T^4$. At lower temperatures all the radiation is concentrated into the three p-wave channels (the lowest order possible for EM radiation) and the rate of cooling is not a function of the surface area and is bounded by our T^2 law of equation (35). The same law holds for any mode of cooling, provided that the relevant wavelengths are long enough. The number of channels is then finite and can be evaluated simply.

If we are prepared to be more specific about the mode of cooling, even more useful formulae can be found. In the instance of cooling by conduction through a metal, the electrons carry away the entropy. If the system has surface area A, the number of channels is given by the number of allowed electron wavevectors parallel to the surface that also correspond to some point on the Fermi surface

$$N = [2A/(2\pi)^2] \sum K_{Fj} \tag{38}$$

where the first factor in square brackets is the density of states and the second is a sum over the areas of separate sheets of the Fermi surface projected onto the plane across which cooling is taking place. The factor 2 is for electron spin. A crude estimate can be made of the Fermi surface area as follows:

$$\sum K_{Fj} \simeq [(2\pi)^2/a](n_v/2) \tag{39}$$

where a is the area of unit surface cell of the crystal and n_v is the number of valence electrons per unit bulk cell, therefore

$$N \simeq n_v A a^{-1}. \tag{40}$$

Defining $\dot{s}$ and $\dot{q}$ to be the rates of flow per unit area we find

$$\dot{s} \leqslant (k_B^2 T/6\pi\hbar) \sum K_{Fj} \tag{41}$$

$$\dot{q} \leqslant (k_B^2 T^2/6\pi\hbar) \sum K_{Fj}. \tag{42}$$

As we saw in § 2, electrons are very efficient at cooling because the separate channels operate at maximum efficiency, provided only that the relevant section of the Fermi surface is more than $k_B T$ away from a band gap. Maximum transfer of heat occurs when electron scattering mechanisms in the metal are negligible. Then there is a 'convective cooling' by electrons which can be extremely efficient. For example, in a typical metal

$$k_F \simeq 2 \times 10^{10}\ \mathrm{m}^{-1} \tag{43}$$

and

$$\sum K_{Fj} \simeq \pi k_F^2 = 4\pi \times 10^{20}\ \mathrm{m}^{-2} \tag{44}$$

hence

$$\dot{q} \leqslant 10^8\ T^2\ \mathrm{W\ m}^{-2}. \tag{45}$$

This cooling rate contrasts strongly with the maximum cooling achieved in computers, $2 \times 10^5\ \mathrm{W\ m}^{-2}$ (Russell 1978).

As an example, take a 10^2 W laser beam focused to a 1μ spot on a metal surface. The minimum temperature attained by the metal, assuming that the electrons are responsible for cooling, is

$$T \geqslant 10^{-4}\sqrt{\dot{q}} = 10^4\ \mathrm{K}. \tag{46}$$

As an illustration of how much less efficient the electron system becomes in the presence of a gap, take the case of an intrinsic semiconductor with gap E_G. For this special case combining equation (32) with equation (30) gives

$$\dot{q}_+ \leqslant (k_B E_G/8\pi^3\hbar) A_{BZ} T \exp(-E_G/2k_B T) \tag{47}$$

where A_{BZ} is the area of the occupied Brillouin zones projected onto the plane across which heat flows. Thus, for a gap of 2 eV the electrons reach 10^3 K at a much lower laser power of about 1 W.

4. Conclusions

The concept of channels of information or entropy flow through a medium has been developed and the fundamental inequality between entropy and energy flow developed explicitly for various media which closely approximate a wide range, not only of information transfer systems, but also of heat-flow systems. I suspect that the inequality as applied to a single channel holds completely generally.

It has been argued that communication systems operate on a finite number of easily identified channels to each of which our inequality separately applies. In the more general case of entropy flow, the number of channels can sometimes be easily identified, especially when the cooling medium is a component of a solid. The number of channels is proportional to the surface area and bounds for the entropy and energy flow through unit area of surface can be found in many cases.

Acknowledgments

I would like to thank Rolf Landauer for much assistance with many parts of this paper; IBM for their hospitality during the summer of 1982 when some of this work was performed, and BP Venture Research for general support of this project.

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